THE ECONOMIC DEFINITION OF ORE

Kenneth F. Lane

CUT-OFF GRADES IN THEORY AND PRACTICE

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First published 1988; Reprinted with minor corrections 1991.

Reprinted with updates and improvements 1997.

This 4th edition printed February 2015

Grateful acknowledgement to Dr John Bensley, Richard Wooller and Chris King (graphics) who converted the pre-electronic edition, holding true to the original version as much as possible and making the changes from the author.

ISBN 978-0-9941852-2-8

Contents

Acknowledgements

Many colleagues in the mining industry, particularly within Rio Tinto, have been associated with the work which is described in this book. I should mention especially Mike Blackwell, the engineer who nursed the infant ideas to maturity on their first major application at Bougainville, and also Allen Sykes, who signposted the straight and narrow economic path.

I have collaborated with RTZ Technical Services on many applications and new developments. Over the years many members of the staff have contributed enthusiastically and given constant encouragement. More recently, Metalica Consultores in Chile (particularly Juan Camus) have been keen supporters and their help with the promotion of a wider understanding of the subject has been very valuable.

* * *

The book has been out of print since 2010 and I assumed that interest in the subject had waned, although occasional enquiries filtered through to me. Now, in 2014, I learn that this assumption was premature and that the book is being reprinted on the initiative of Brett King, a friend and colleague for many years. This revival means a lot to me and I very much appreciate his considerable efforts together with those of other colleagues still working to promote a better understanding of the economics of mining.

Kenneth F. Lane

Foreword

(to the first edition, 1988)

I believe this book is important for three reasons.

First, the topic is important and, although widely researched and taught in mining schools, no authoritative book on the subject has been written.

Second, it is written in a clear manner, which will help even the non-mathematically inclined student or the professional associated with the development of mines.

Finally, with the strong emphasis given to the economics of the whole mining process, and not just the economics of orebodies, the author drives home a lesson often forgotten by the geologist and even the mine planner.

Ken Lane has done a service to the mining industry in writing this book. His early pioneering work some 20 years ago in the RTZ Group brought a new dimension to the world of mining, and I am glad that he has continued his association with the RTZ Consulting Group, which benefits greatly from his advice.

Sir Alistair Frame

Chief Executive of Rio Tinto Zinc Corporation

Present Value Analysis

Time	T	Short interval	t
Resource Available	R	Small increment	r
Variables defining		Time per unit of resource	τ
Exploitation Strategy	Ω	Exploitation Strategy for t	ω

Present Value	$V = V(T, R, \Omega)$ (also W)	
Maximum P.V.	$V^* = V^*(T, R)$	
Opportunity Cost	$F = \delta V^* - dV^*/dT$	Primes(') indicate special values in context
Cash Flow	C per year	
	c per unit of resource	
Increment in P.V.	v per unit of resource	
Cost of Capital	δ (100 δ %)	
Terminal Value	Γ	

Economic Model

	Throughput	Variable Cost (/unit throughput)	Capacity (throughput/year)
Mining	Material	m	M
Treating	Ore	h	H
Marketing	Mineral	k	K

Fixed or Time costs	f per year
Price	p per unit of material
Cut-off Grade	g mineral/unit of ore
Optimum Cut-offs	G mineral/unit of ore
Average Grade	$\bar{g}$ mineral/unit of ore
Ore/Material Ratio	x
Yield during treatment	y (100y %)
Quantities	q
Stockpile recovery cost	s per unit of material
Stockpile size	S units of material
Cut-off intercepts	Υ_1, Υ_2 mineral/unit of ore

Suffixes denote years or have particular significance in the context. Certain other symbols are also used with strictly local definitions.

CHAPTER ONE

Introduction

When I first joined the mining industry I believed that ores, like buried treasures, possessed immediately recognisable characteristics; they glowed in the dark, or glittered in torchlight, or were black and strangely heavy. This is a preconception which is shared by most laymen and I was effectively a layman at the time since my training had been in mathematics and economics, not in mining.

Even after working at several mines, my conviction was unshaken. When the geologists pointed to rich seams in the hanging walls or chipped what they proclaimed as good ore samples from new headings, I attributed my inability to distinguish the material from any of the surrounding rock to my shameful ignorance of mineralogy. I was impressed; these people were sensitive to subtle differences which no amount of concentration revealed to me.

Then, inevitably as a mathematician, I became concerned with the statistics of sampling. I learned that samples are the nerves of operating mines, borehole samples, chip samples, channel samples, grab samples, dust samples — thousands of samples which are regularly taken, assayed, plotted and interpreted. They are the means of mine control and, in the grade control department, they are the means for determining the limits of the ore. At last I understood. Although some ores may be distinguishable by certain physical properties, ores in general are defined operationally by a cut-off grade; material with a mineral content above the cut-off is scheduled for treatment, other material is left or dumped as waste.

Having made this discovery, I asked what seemed to me to be the obvious next question:

Why work to this particular value of cut-off grade rather than some other value?

Typical answers were:

We have always worked to 0·3%.
Head office decided 5% combined metals some years ago.
That is a technical matter; we leave it to the people on-site.
I think several cut-offs were examined in the feasibility study and 1% seemed best.
I guess our costs are running at $10 a tonne and uranium is worth $10 a pound, so 1 lb/tonne must be about right.

The inadequacy of these answers stimulated my original interest in the definition of ore. The fact seemed to be that the subject was not clearly within the scope of any one of the industry's professions — mining, mineral processing, geology or economics — and, as a consequence, had not received the attention which it deserved. I could find no authoritative references, only passing mentions in text books and a few papers. It seemed ironic, in an industry devoted to mining ore, that its definition of ore should be so taken for granted.

This was over 30 years ago. Since then I have worked on mine planning and economics of cut-off grades for very many mines around the world including most of the large scale operations, both open pit and underground. In the course of this experience, a theoretical basis for the definition of ore has been established and the theory has been developed to apply to most methods of mining. The concepts are widely accepted and many groups have adopted the techniques as standard practice for the determination of cut-off grade policies. A recent example is Codeleo, the Chilean national copper company. The applications, particularly to the design of big mines, have achieved substantial improvements in the overall economics.

* * *

Minerals permeate the earth's crust in varying concentrations around the world. A shovel full of soil from most gardens will probably contain measurable amounts of aluminium, silica, potassium, iron, etc. These may be of interest to the gardener but not normally to the miner for the very obvious reason that the concentrations of the mineral are too low.

In fact, concentration is the critical property. The mining industry can be regarded as an industry whose whole concern is with concentration — the progressive concentration of minerals to a form where they become marketable.

Typically the process proceeds in stages. The first stage is exploration, which is the search for mineralised regions in the earth's crust where some degree of natural concentration has already occurred. The second stage is extraction, in which certain parts of a mineralised region are recovered for further treatment. Succeeding stages are treatment stages such as crushing, grinding, flotation, leaching, smelting and refining.

The subdivision of the whole process into stages, and the locations of the exact boundaries between them, is dependent upon the economics of the technologies involved. For example, finer grinding will usually result in better yields of mineral from the ground material but at the expense of higher power consumption. Similarly, better yields will also usually result from longer residence times in leach circuits but at the expense of higher acid consumption. Thus the stages interact and the optimum combination can only be decided with reference to the whole operation.

This book is concerned exclusively with the second stage, extraction. It is often referred to simply as mining, although references to the mining industry are taken to embrace all the stages. The boundary, which is the main focus of attention, is the one which distinguishes the material within a mineralised body that is to be extracted and treated from the remainder. This boundary is commonly specified by a cut-off grade and the theory which is developed in the following chapters concentrates on such cases. In other cases, where for example blending is necessary to meet some quality requirement, the theory is no longer directly relevant.

To avoid ambiguities, the word ore is used solely to describe the material which is extracted for treatment. In other words, by definition, mines extract ore. Hence, establishing an economic basis for determining cut-off grades is, in effect, providing an economic definition of ore.

The form of the presentation is first to enunciate the economic principles which are relevant to the analysis and then to trace the consequences of applying these principles in various circumstances. Although this leads to mathematical complexities in some areas, it transpires that clear and consistent thinking is required more often than agility with algebra. During the course of the work a coherent theory of cut-off grades is developed and the derivations of the main formulae are described in some detail. The intent, however, is to explain the ideas rather than to achieve mathematical rigour. The theory is illustrated with practical case studies which are based upon adaptations of actual applications. Sufficient material is included for the case studies to serve as useful references.

At the outset, problems of semantics are encountered. Many important words are commonly used casually in the industry with different meanings in different contexts. This leads to misunderstandings and has certainly inhibited the development of clear concepts. The word "ore" has already been mentioned and is a prime example. Mineralised bodies are called orebodies before they are mined and even before any serious plans to mine them have been mooted. Then reserves of possible, probable, proven, drilled, inferred and developed ore are often quoted when, strictly speaking, they are estimates of tonnages of mineralised material which could be ore under certain circumstances in the future. In this text, care is taken to avoid such inconsistencies of definition. This sometimes leads to the unfamiliar usage of certain terms, but the intention is not to be pedantic. Only when a distinction is important are abnormal words and usages adopted.

Further problems arise from the finite nature of mineralised bodies. Mining operations based upon them must be of limited duration, a feature which introduces complications into the economic analysis. Unfortunately, it can also introduce an emotional charge into personal attitudes on the subject. These, aggravated by the semantic difficulties, too often result in unedifying arguments rather than useful discussions. Here, problems of this kind are avoided by the strict adherence to the logical consequences of assumptions and objectives. The aim is the development of a definition of ore that is optimum according to accepted current economic ideas, uncompromised by other considerations.

Optimum economic cut-off grade policies have been calculated for a variety of projects over the past 30 years. Broadly, the conclusion in the case of mines which are well established is that these optimum policies seldom differ much from current practices. The reason for this is that the mines have usually been designed, and subsequently perhaps modified, to handle the quantities of ore and mineral and the associated grades to which these practices give rise. The capacities of the equipment and the installations do not often permit much flexibility and therefore cut-off grades can only be varied within narrow limits. In contrast, when expansion schemes are being designed; and even more so when totally new mines are being developed, the theory can indicate cut-off grades quite different from conventional policies with very substantial corresponding improvements in the overall returns.

Although a surprising number of cut-off grade calculations can be done by hand, they can become very intricate. This observation applies particularly to the determination of cut-off policies for long-range planning. In this book these calculations are performed with a computer program which has been developed for the purpose over the years. It is called OGRE (Optimum Grades for Resource Exploitation). The rights to this program are owned by the Rio Tinto group but other software packages are available. One of the most powerful is marketed by COMET Strategy in Australia.

CHAPTER TWO

Economic Principles

Inevitably, the question that is asked about any body of mineralisation is— Does it contain ore? Or, more strictly — Does it contain any potential ore?

An inconvenient consequence of adopting an economic definition of ore is that there is no longer any inherent property of the mineralised material which permits an answer to this question in isolation. Although exploration personnel often calculate a 'dollar value per ton of rock' in order to assess targets, in fact, minerals in the ground have no explicit value. Not until they have been extracted, treated and delivered to a customer is any value realised. Therefore, the economics of ore definition cannot be assessed separately from the economics of the total mining process. Indeed, it is the economics of the mining process which determine the economic definition of ore.

This point is fundamental. Mineralised bodies are often referred to as valuable resources. In a sense they may be but regarding them as such can be misleading. They are certainly not a valuable resource that might be compared with cash in a bank or even a crop on the ground. The only immediate value they could possess is the price a mining company might bid for the right to mine them. More realistically, a mineralised body should be regarded as a possible opportunity for development, the development being, of course, a mining operation. Any value that might be ascribed to the mineralisation is then realised as an integral part of the proceeds of the operation.

It follows from these considerations that, in order to establish an economic basis for ore definition, the analysis must first be directed to an operating mine. An understanding of the economic factors which influence ore boundaries must be derived from an understanding of the economic factors which influence the whole mining process.

The factors concerned are many and include markets, prices and costs, but they can be integrated using the economic concept of value. A mining operation earns revenue and incurs costs; it is therefore an economic entity and an estimated value can be ascribed to it.

This value is clearly dependent upon the definition of ore, some bases of definition giving rise to higher values than others. The basis which generates the highest value is optimum and this basis establishes the economic definition of the ore. In other words, material from the mineralised body should be scheduled for mining as ore if, and only if, the decision to treat it adds to the overall economic value of the operation. This is the crucial criterion.

Economic value estimates are derived from projected cash flows. As a result of earning revenues and incurring costs year by year, an operation generates annual net cash flows. These can be amalgamated into a value, strictly a 'present' value, by discounting future flows back at an appropriate cost of capital and totalling them. The theory of present values — or their inverses, internal rates of return — and methods for determining the cost of capital are beyond the scope of the present book. The theory has been widely discussed and analysed in many books and papers and it has been accepted in the mining industry, certainly by its financial analysts. It is almost universally used for valuing properties and evaluating new projects. However, its use as a means to determine an optimum operating policy is less common.

Of course, this does not make it any the less valid, but the unfamiliarity has contributed to scepticism about the present value criterion in this context, particularly when the results differ significantly from conventional ideas. The differences too, are usually that present value maximisation indicates higher grades and higher rates of mining which seem inconsistent with trusted conservative mining policies.

The expression of reservations about the present value criterion, though, is far from proposing an alternative. In fact, alternatives are rarely formulated unambiguously, but the two most general contentions are that:

1) mineralised material should be treated as ore if it will provide a contribution to profit;

2) mining should be conducted in such a way as to maximise the extraction of valuable mineral.

The cut-off policies which result from the application of these criteria can be the same, depending upon the definition of the terms employed, but they are discussed separately.

The first criterion in some form is popular among technical staff. The question of what constitutes a contribution to profit is the subject of much debate, however. It is often argued that any material for which the value of the recovered mineral will exceed the marginal cost of treating it should be ore. Sometimes a contribution towards overheads is added to the costs and sometimes, beyond this, a minimum profit requirement is also added. The basis of the argument is that if such material is not classified as ore, then an opportunity to earn profit has been wasted.

The flaw in the argument is that it totally overlooks capacities. It is equivalent to arguing that a retailer should add to his stock any goods which promise to yield a marginal profit. Retailers do not do this. They are all aware that space is limited and within this limitation they try to stock the more profitable items. Similar considerations apply to a mine. It has a capacity which is limited by some part of the installation — the shaft, the mill, the truck fleet, the rate of development, etc. — and within this limitation it should choose to process the more profitable material. This policy is consistent with the interpretation of the criterion which includes a minimum profit margin, but the supporters of the criterion usually give no basis for determining the margin, other than company policy. The present value criterion, by contrast, gives a precise basis derived as a trade-off between present and future earnings via the present value function.

The second criterion that the extraction of valuable mineral should be maximised is frequently proposed by mineral rights owners, local governments and conservationists. Of course, it immediately begs a question, what is valuable mineral? An extreme argument is that all the mineral or all the geological reserves (whatever they are) should be extracted in the interests of conserving resources. This is an unrealistic stance which usually stems from a misunderstanding of the way in which minerals are distributed in the ground. A less extreme view is that the mine should be developed in such a way that poorer material is extracted along with richer material in an acceptable blend yielding a satisfactory profit. Of course, every mine blends poorer and richer material of necessity and the point of a cut-off grade is to determine just how poor poorer material can be. The protagonists of the maximum extraction criteria, however, usually imply a degree of subsidy for poor material which would not be economic on its own. What this means is unclear, but the idea of cross-subsidies of ore grades is economically unsound except in special circumstances. A more reasonable view defines valuable material in the same way as in the first criterion. In this case the two give the same result and suffer from the same objection about the effects of capacity.

Both of the criteria have another major shortcoming; they do not deal satisfactorily with price variations. Nor do they deal satisfactorily with variations in other economic parameters, but price is the predominant influence.

The point is that, as with all break-even calculations which compare inherent value with cost in some form, higher prices lead to lower cut-off grades. Now, lower cut-off grades yield lower average grades and if the quantity of ore treated remains the same, as it most often does, the output of mineral declines. This is quite the reverse of what should happen in the market. Higher prices imply a deficiency of supply in relation to demand and should prompt an increase in supply, not a decrease. Further, the mine itself is in the position of selling less at the higher price than the lower, a policy which cannot make sense.

Present value is the only criterion which does incorporate a means for dealing with varying economic conditions. Parameters defining the conditions are included in the present value estimates and affect the optimum cut-off calculations in a way which avoids nonsensical reactions to price changes. This feature is illustrated in Case Study 2 (p.112).

Many of the criticisms of the present value criterion are actually expressions of special interests. For example, it is rarely advantageous for the staff of an operating mine to support a policy which shortens its life; they would only be threatening their own livelihoods. Similarly, local governments usually wish to see industrial activities prolonged because this entails continuing employment, continuing taxes and, perhaps, continuing royalties.

There is an apparent conflict here between the interests of the contributing parties but, from a strictly economic point of view, it is not an inevitable conflict. If a mine is planned in such a way as to maximise its net present value (excess of present value over capital costs) then, in theory, there is more wealth to share between the participants. Everyone could be better off. Whether, in the event, they are or not depends upon the nature of the agreements between them, but this is a huge subject in its own right.

To repeat, this book is concerned wholly with the economic definition of ore as that definition which maximises the net present value of a mining operation. There may well be reasons, in special cases, for adopting other bases of definition, necessarily suboptimum, but such cases are not covered in this book except for occasional incidental references.

CHAPTER THREE

Finite Resources and Present Values

As has been stressed already, every mine is established on a body of mineralisation which is ultimately of limited extent. Some are very localised and are mined out in a matter of months; others are vast with seemingly endless sources of ore. Nonetheless, they are actually finite and, sooner or later, will be depleted.

This characteristic makes the analysis of operating strategies for mines very different from the analysis for most other industrial or commercial undertakings. The fundamental concept of optimisation by maximising present values is just as relevant. However, other undertakings are not usually based upon an exhaustible resource and, hence, current operating strategies do not react on the future in the same way. For a mine, higher mining rates will shorten the life and vice versa. The effects of this must somehow be built into the analysis. It is, of course, the present value function itself which provides the means for making cash effects, which occur at different times, commensurate. An essential preliminary to an analysis of cut-off strategy is, therefore, an examination of present value maximisation for an operation based upon a finite resource. This necessarily involves some mathematics but detailed explanations are also given in the following paragraphs.

Denote the present value of an operation based upon a finite resource by V. The operation could be a mine but the analysis is general and could apply to any type of finite resource operation like, for example, the liquidation of a stockpile. Vis calculated as the total of the future cash flows discounted back to the present. If these cash flows, year by year, are C_1, C_2, ... and the cost of capital is δ ($100 \times \delta$ as a percentage) then

$$V = C_1/(1+\delta) + C_2/(1+\delta)^2 + \ldots$$

The cash flows C_1, C_2, ... are dependent on the prices and costs prevailing at the time and therefore the value of V itself is dependent upon the present time, T, which forms the base of the calculation. In other words, the present value of two exactly similar operations calculated at different times will, in general, differ.

$$\text{i.e. } V = V(T)$$

Further, the present value must also depend upon the amount of resource, R, still available. In general, it must decline as the resource is consumed and fall to zero when the resource is exhausted.

$$\text{i.e. } V = V(T,R) \text{ and } V(T,0) = 0$$

This, however, is not the end of the story. V must depend upon many more variables which describe the way in which the operation is to be conducted. Rather than writing a long list, a convenient mathematical convention is to represent these variables by one symbol. Call it Ω. Ω defines the *operating strategies* to be employed in the future and

$$V = V(T,R,\Omega)$$

In the case of cut-off strategies for a mine, Ω would consist of the variable cut-off (g, say) which can take on different values, g_1, g_2, g_3, ... for the remaining years of the mine 's life. Such a sequence of values can be called a policy and therefore g_1, g_2, g_3 ... define a cut-off policy. Thus, in this case, if cut-off grades are the only parameters being investigated,

$$\Omega = g_1, g_2, \ldots$$
$$V = V(T,R, g_1, g_2, \ldots)$$

Now, reverting to the general case, of all the sets of operating strategies, Ω, which could be adopted, there must be one set, at least, which is optimum in the sense that this set gives rise to the maximum value for V(T,R,Ω). Or, to put it another way, any set of values for Ω will give rise to some value for $V(T,R,\Omega)$, and if Ω is varied over every conceivable set of values, $V(T,R,\Omega)$ will vary correspondingly, and one, at least, of the values it assumes must be maximum.

Call this maximum $V^*(T,R)$. A significant observation at this point is that it is no longer a function of Ω. $V(T,R,\Omega)$ itself was, but $V^*(T,R)$ is not.

This is like observing that heights on a map are a function of position but the height of the highest point is not. It has a position, of course, but it is dependent only upon the particular area covered by the map, nothing else.

$$\text{Thus } \mathrm{Max}_{\Omega} \{V(T,R,\Omega\} = V^*(T,R)$$

Before developing the algebra further, consider the function $V^*(T,R)$ more closely. It is a function of two variables and therefore forms a surface. For $R = 0$, when the resource is exhausted, $V^* = 0$ so the surface slopes to zero along the $R = 0$ axis. Also, the function must decrease, in general, as R decreases (only if the optimum strategy involves a cash outflow will it increase).

A possible surface is shown in figure 3.1 and an understanding of this figure is important. The surface it represents consistently slopes down towards the lower axis on which the resource available is zero. This axis is itself the zero 'present value' contour. Moving across the figure, the surface has ripples in the time direction. These ripples arise from variations in the projections of prices and costs which cause variations in the present value estimates at different times even for the same resource availability. Constant price and cost projections would give a series of horizontal parallel values.

Of the two independent variables T and R, the former is not controllable. It is simply the date on which the present value is based, i.e. it defines the present to which cash flows are discounted. The latter, R, or more exactly the rate at which R is decremented, is the variable which is directly affected by operations.

Of course, as R is decremented (that is, as resource is consumed), time necessarily progresses and the changing resource level with time can be graphed. It forms a line or track as is illustrated in figure 3.2. This track defines the rate at which the resource is consumed at every

Figure 3.1 Maximum Present Value Surface

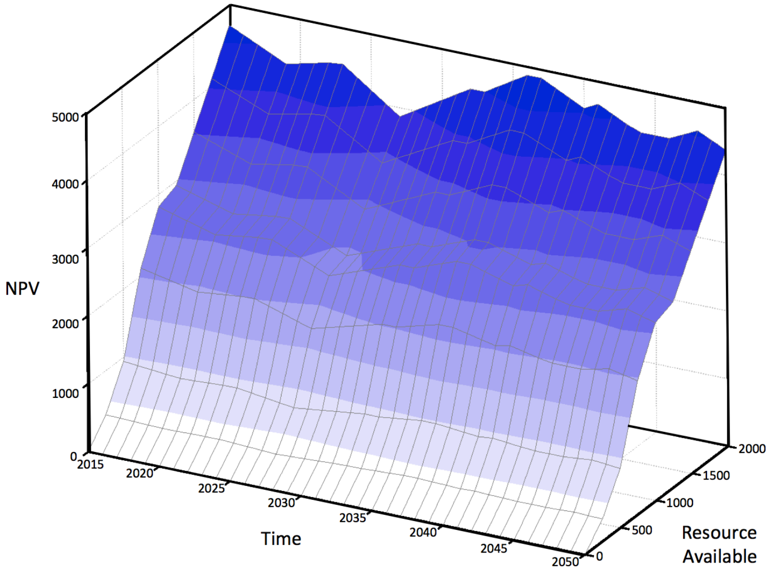
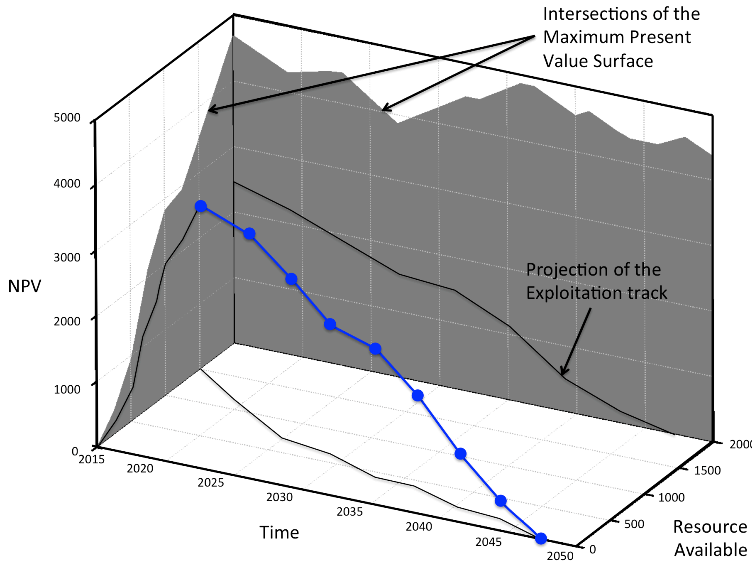


Figure 3.2 An Exploitation Track



CHAPTER SEVEN

Effective Optimum Cut-Off Grades

The quest for an optimum has led to a plethora of cut-off grades – six, in fact. There are three limiting economic cut-off grades and three balancing cut-off grades corresponding to the three possible pairings of the limiting components of the mining system.

The multiplicity of contenders for the title of effective optimum is, of course, a consequence of the form of the economic model. It gives rise to six different possibilities, but only one of them is feasible under any given set of operating conditions. Frequently, the feasible possibility is apparent from the structure of the application, but this is not always the case and a logical procedure for identifying the right one is required.

The best way to examine the interrelations of the six cut-offs is to calculate the variable v which was introduced in Chapter 5. This variable v is actually the rate of change of V*, the optimum present value, with respect to resource usage (dV*/dR). In other words, it is the increment in present value per unit of resource utilised. It has to be maximised in order to determine the optimum cut-off grade. The formula is

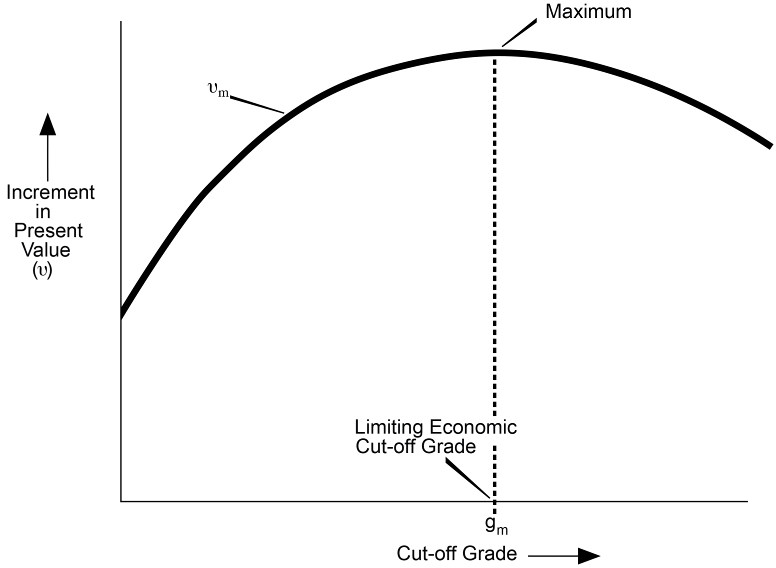
$$v = (p - k)xy\bar{g} - xh - m - (f + F)\,\tau$$

v takes three forms according to the determinant of τ

$$v_m = (p - k)xy\bar{g} - xh - m - (f + F) / M$$
$$v_h = (p - k)xy\bar{g} - x\{h + (f + F) / H\} - m$$
$$v_k = \{p - k - (f + F) / K\}xy\bar{g} - xh - m$$

The graphs of all three forms of v as a function of the cut-off grade, g,

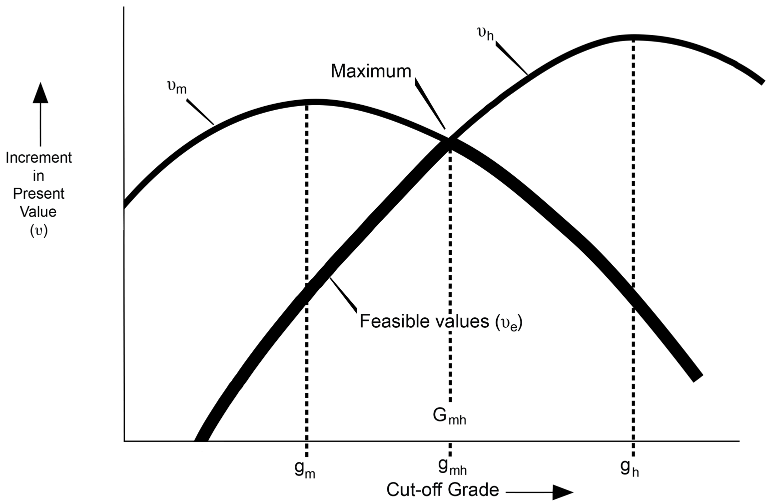
Figure 7.1



Increment in Present Value versus Cut-off Grade

Single Component (M); Limiting Economic Optimum

Figure 7.2



Increment in Present Value versus Cut-off Grade

Two Component (M & H); Balancing Optimum

are similar; convex upwards with a single maximum. This maximum

CHAPTER TEN

Calculating a Complete Cut-Off Policy

Chapters 5, 6 and 7 were concerned with the calculation of the optimum cut-off grade at a single point in time. This chapter is concerned with the calculation of a sequence of optimum cut-off grades over an extended period. This constitutes what has been defined as an optimum cut-off policy.

No new principle is involved. Every cut-off grade in the sequence is an optimum cut-off grade calculated according to the principles described earlier. However, they have to be consistent in the sense described in Chapter 3; the corresponding annual cash flows and the associated sequence of present values must conform to a consistent definition of present value throughout the whole period.

In the terminology of Chapter 3, an optimum cut-off policy corresponds to a complete optimum exploitation strategy for the resource-in this case, the mineralised body. The problem is to find an exploitation track along which a consistent associated sequence of present values with a given terminal value is achieved. This is then an optimum exploitation track and the present values at every stage along it are maxima.

Usually, the period for which a policy is to be determined is the remaining life of the operation. In this case, the terminal present value is zero. It is possible to calculate a policy for a shorter period but, in such cases, a terminal present value must be specified for the resource which will remain after termination. The reason for this is that the optimisation process maximises the present value at a certain level of resource, and the maximisation formulae necessarily incorporate a value associated with continuing operations based upon any resource remaining beyond the exhaustion of the increment under study.

Obviously, the calculation of a sequence of optimum cut-off grades requires a database of information for the complete period. This means annual forecasts of the economic parameters as described in Chapter 8, and mineralised reserve estimates as in Chapter 9.

The main problem in calculating a cut-off policy as opposed to a single optimum cut-off grade is when and how to start. A tempting approach is to create the policy in reverse. Starting with the terminal value, an optimum cut-off can be calculated; hence, a cash flow for the final year and, from this in combination with the terminal value, a present value for the year prior to termination. This figure can then in turn be used for the penultimate year and the present value calculated for two years prior to termination. And so on ...

However, there is a difficulty with this approach. The cut-off policy affects the rate of progress through the mineralised body and, until the policy has been determined, the termination time is unknown. The difficulty could be overcome by taking a series of termination times but it is more logical to start at the beginning.

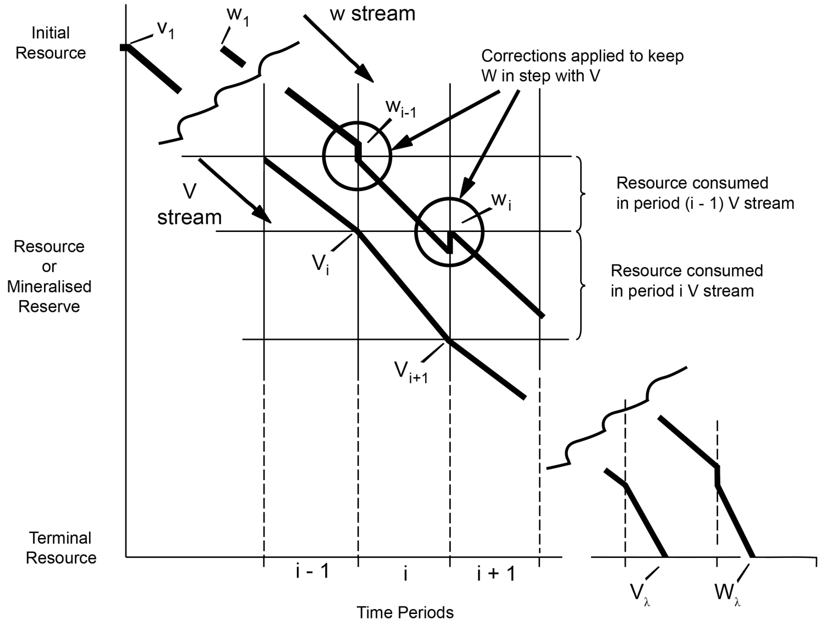
The problem is then how to start because the initial levels of present value are unknown. This problem can be surmounted, however, by a mathematical iteration process. Initial levels are assumed, a policy calculated, and the present values on termination compared with the specified terminal value. Depending upon the difference, the initial levels are modified and a new policy calculated. This is a form of mathematical gunnery practice in which the direction of the gun barrel, defined by the initial levels of present value, is progressively modified until the shot is close to the target, the specified terminal value.

A complication arises from the requirement to follow two sequences of present values. As was shown in Chapter 3, the formula for determining the optimum strategy at each stage incorporates the opportunity cost term

$$F = \delta V^* - dV^*/dT$$

dV^*/dT is the rate of change of present value with time and an

Figure 10.1



Graphical Representation of a Single Optimum Policy Iteration

Subtracting them from V_1 and W_1 respectively gives new initial levels for another iteration.

This calculation can be elaborate and time consuming and it is best performed on a computer. It is incorporated into the OGRE (Optimum Grades for Resource Exploitation) program and in another form into COMET. Computer systems such as these are very powerful tools for studying cut-off policies. They permit the investigation of the factors which affect the policy and the way in which they influence policy over the course of time.

Illustrations of the calculation of cut-off policies are included in Case Studies 4, 6 and 7.

CHAPTER ELEVEN

Parametric Cut-Off Grades

A cut-off grade is defined as parametric if it is only indirectly related to the grade distribution of the mineralised body. Parametric cut-off grades are not at all uncommon and arise from a variety of causes including — recoveries which vary with different types of mineral, the presence of minor minerals whose equivalent values are simply added to the main mineral of the body and also to inaccuracies in grade control.

Take as an example a copper mineralised body which also contains some molybdenum. The original grade categories may well have been defined in terms of both copper and molybdenum but, because the molybdenum is only of minor importance, the complexities of a two dimensional grade analysis are avoided by calculating the copper equivalent of the molybdenum in each copper category and adding this to the copper content. A typical calculation is illustrated in Case Study 4 (p.119).

This involves a compromise between accuracy and practicability, of course. In theory, either the two dimensional grade distribution should be retained and the analysis conducted according to the methods described later in Chapter 17 or the reserve should be recompiled on the basis of the combined minerals. The latter entails adding one mineral to its equivalent of the other to form the combination at some earlier stage in the compilation of the mineralised reserve. The precise stage depends upon the method of reserve estimation: it could be the original sample stage or a later block estimate stage. In either case, recompiling the reserve is a major arithmetical exercise and, moreover, it is one which may have to be

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